Rec Math SIGMAA Happy Hour

Mathematical card tricks learned from Ron Graham & Persi Diaconis

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Ron Graham (1935-2020): Baby Hummers

> *"While not Ron's trick, the one I saw him do the most was the 6 card Baby Hummer."*
>
> Steve Butler

Charles Hudson's 4-card variation on a Bob Hummer classic from the early 1940s is the first trick in the wonderful book *Magical Mathematics: the mathematical ideas that animate great magic tricks* (Princeton, 2012), by Persi Diaconis & Ron Graham.

We start with this "baby Hummer", and then explain modifications which allow for a 6-card version.

Start by mixing up any 4 cards face down, and then peek at and remember the bottom card. (Later we'll do the same with a packet of 6 cards.)

4-Card Baby Hummer (by Charles Hudson, 1967)

Step 0: You know the bottom card in a face-down packet.

Step 1: (C1) cut the top card to the bottom.

Step 2: (T1) turn the top card over.

Step 3: (Cn) cut any number of cards from top to bottom.

Step 4: (T2) turn the top TWO cards over, as a single unit.

Step 5: ("CATTO") repeat Steps 3 & 4 as often as you like.

Step 6: (T1) turn the top card over.

Step 7: (C2) cut the top TWO cards to the bottom.

Step 8: (T1) turn the top card over.

Fan the packet of cards: 3 will face one way, and the other card is the original bottom card which was peeked at.

A fave trick of Ron Graham: 6-Card Baby Hummer

Step 0: You know the bottom card in a packet of 6 cards.

Step 1: (C1) cut the top card to the bottom.

Step 2A: (T1) turn the top card over.

Step 2B: (C2) cut the top TWO cards to the bottom.

Step 2C: (T1) turn the top card over.

Step 3: (Cn) cut any number of cards from top to bottom.

Step 4: (T2) turn the top TWO cards over, as a single unit.

Step 5: ("CATTO") repeat Steps 3 & 4 as often as you like.

Step 6: (T1) turn the top card over.

Step 7A: (C2) cut the top TWO cards to the bottom.

Step 7B: (T1) turn the top card over.

Step 7C: (C2) cut the top TWO cards to the bottom.

Step 8: (T1) turn the top card over.

Fan the packet of cards: 5 will face one way, and the other card is the original bottom card which was peeked at.

Bob Hummer's "Face-up/Face-down Mysteries" (1942)

Underlying the baby Hummer tricks is the following fact. Start with any even-sized packet of face-down cards, let's assume we have 10.

Do the CATTO (or CATO) move repeatedly:

Cut any number of cards from top to bottom

And

Turn the top TWO cards over, as a single unit.

No matter how many times you do these, some structure remains:

The number of face-up cards at positions (1, 3, . . . , 9) is the number of face-up cards at positions (2, 4, . . . , 10).

Bob Hummer's "Face-up/Face-down Mysteries" (1942)

Application: after doing CATTO over and over to a 10-card packet, *flip over the cards in the even positions*. (This can be done with the cards held under the table or behind your back.)

There will be exactly 5 cards face up, and 5 face down.

Even better, the face-up ones started in positions of the same parity.

(Even best if the face-up ones started in the even positions.)

Start with two intertwined poker hands, say 3 to 7 of Diamonds in the odd positions, and 10 to Ace of Clubs in the even positions. After many CATTOs, and flipping cards in the even positions, bring the packet forward and fan to reveal the impressive flush in Diamonds. As soon as the audience is "suitably" impressed flip over the remaining 5 cards to reveal the winning flush in Clubs.

Abra-ka BABA ABBA (Persi Diaconis inspired)

Persi Diaconis has been demonstrating the mathematical principle behind our final trick to lucky audiences around the world for over 30 years. He refers to it as "universal cycles".

Pioneers from the past include Bob Hummer, Larsen & Wright, Ron Wohl, Max Maven and Leo Boudreau. The roots of the subject go back to Charles Jordan's "Coluria" from 1919.

Persi has been known to perform it with a packet of 32 thick cards; nobody suspects that he's not playing with a full deck.

After some false shuffles (yes, the packet is pre-arranged) he hands out the cards requesting that somebody cuts the packet, and passes it to a second person, who also cuts then passes the cards to a third person. This continues until a fifth person cuts the packet.

Abra-ka BABA ABBA (Persi Diaconis inspired)

The fifth person takes the top card, and passes the rest of the packet back to the fourth person, who takes the new top card, and passes what is left to the third person, and so on.

Finally 5 people have a card each, which they look at.

(Yes, those 5 cards were side by side in the original packet.)

Persi says he is going to identify each card, then pretends that he isn't thinking clearly due to lack of coffee. He needs a little help.

Can the people with Red-faced cards please raise their hands?

Arrmed with this seemingly inconsequential information, he soon names all 5 cards correctly to thunderous applause.

A fave trick of Persi Diaconis: de Bruijn sequences

What makes this work is the existence of a 32-term sequence of 0's and 1's with the property that when wrapped around to yield a circle, a sliding window of length 5 will encounter each possible length 5 string (such as 00000, 01011, 11101, etc) exactly once.

These are *de Bruijn sequences*, and they exist for any power of 2.

Persi remembers such a 32-term sequence, with 0 corresponding to one colour and 1 to the other, and more amazingly he remembers a matching sequence of 32 card values!

Once he knows the colour sequence for the 5 cards, he knows where those were in the original packet, and he also knows their suits and values. It's a real *tour de force*.

Over Zoom, a crib sheet could easily be used to make such a performance conceivable for mere mortals.

Scaled down in Sweden

Let's settle for using a de Bruijn sequence of length 16,

0 0 0 0 1 1 1 1 0 1 1 0 0 1 0 1

For cards, we could use the same 16 cards from 3 different decks, to yield a packet of size 48.

If 0 represents Red and 1 respresents Black, and we insert gaps after each 4 cards, we get

RRRR BBBB RBBR RBRB

which can, in turn, be morphed (topologically) to

AAAA BBBB ABBA ABAB.

If you can remember 16 such cards, you can now do such a trick.

References

Persi Diaconis & Ron Graham: **Magicical Mathematics: the mathematical ideas that animate great magic tricks**, Princeton University Press, 2012, winner of Euler Prize 2013.

Card Colm: **"Many Fold Synergies?"** (MAA Online, Feb 2006)

Card Colm: **"What's Black and Red and Red All Over?"** (MAA Online, Dec 2008)

Matt Baker: **CATO (Part 1 of 2)** (Sep 2020) https://www.vanishingincmagic.com/blog/CATO