A Card Trick Learned from John H. Conway

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John H Conway's "The Five Tenacious Boys" Set-Up

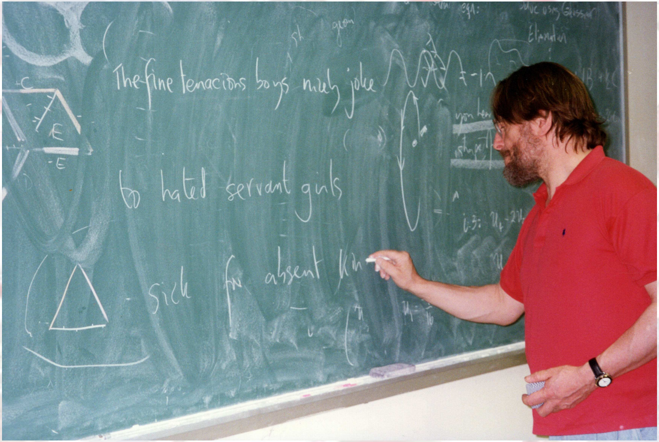
In April 1995, during his Math Awareness Week visit to Spelman College, John Conway entertained us all with a run of card tricks, based on a rigged deck of his own divising. He would genuinely shuffle once, before performing miracle after miracle to the amazement of onlookers.

He taught me the set-up, and for years I'd carry a rigged deck with me to meetings. When we'd spot each other he'd casually say "Does anybody have a deck of cards? If so, then I'd like to show you something." He would then dazzle everyone with the deck which I "just happened" to have with me.

John H Conway's "The Five Tenacious Boys" Set-Up

The sequence was based on a mnemonic of his:

The Five Tenacious Boys Nicely Joke To Hated Servant Girls Sick For Absent Kings.



John H Conway's "The Five Tenacious Boys" Set-Up

This represents the cycling 14-term sequence:

3, 5, 10, Ace, Jack, 9, Joker, 2, 8, 7, Queen, 6, 4, ( ), King (Absent is absent!)

He needed Jokers. He started with a spelling routine which worked for him, but oddly didn't work for those who tried to copy it. Then he'd jettison the Jokers and move on to more mathematical effects.

Suits must also cycle, and we recommend **CLUBS, HEARTS, SPADES, DIAMONDS**.

John H Conway's "The Five Tenacious Boys" Set-Up

Cycling both the value sequence **3, 5, 10, A, J, 9, 2, 8, 7, Q, 6, 4, K** and the suit cycle **CLUBS, HEARTS, SPADES, DIAMONDS** yields:

3C, 5H, 10S, AD, JC, 9H, 2S, 8D, 7C, QH, 6S, 4D, KC, 3H, 5S, 10D, AC, JH, 9S, 2D, 8C, 7H, QS, 6D, 4C, KH, 3S, 5D, 10C, AH, JS, 9D, 2C, 8H, 7S, QD, 6C, 4H, KS, 3D, 5C, 10H, AS, JD, 9C, 2H, 8S, 7D, QC, 6H, 4S, KD

John H Conway's "The Five Tenacious Boys" Trick

For a deck thus arranged, into interwoven 13-value and 4-suit cycles, deal off about half of the cards to the table, thereby reversing their order. Riffle shuffle those into what is left.

If the top two cards are removed, they are sure to be one Red and one Black. If the top four cards are removed, they are sure to be one of each suit.

If the top thirteen cards are removed, they are sure to be one of each value.

These observations are true thanks to the Gilbreath principle which is discussed later.

John H Conway's "The Five Tenacious Boys" Trick

A multi-stage demonstration of "post-shuffle order" can be given as follows.

Right after the shuffle, take off two cards and show that they are one Red and one Black. Do this three more times, so that eight cards have now been removed. Set those cards aside, face down.

Now take off the next four cards, showing that they represent all four suits. Do this three more times, so that sixteen additional cards have been removed. Set those cards aside too, face down.

John H Conway's "The Five Tenacious Boys" Trick

Take off two more cards to confirm that they are one Red and one black, and add to the other discarded cards. Half the deck remains.

Deal off half of those and give them to somebody, keeping the last thirteen cards for yourself.

Each person can confirm that they have one each of the thirteen different card values from Ace to King.

This should surprise an audience given that the deck was shuffled at the outset.

John H Conway's "The Five Tenacious Boys" Trick

Any 13-cycle works. Conway's is just one option.

A different presentation is to ask for a number less than thirteen to be called out as you shuffle.

Let's suppose somebody says Nine. Casually deal out fourteen or fifteen cards—don't do thirteen or sixteen—in a spread-out row, all face up except for the ninth. You can quickly identify the ninth one. Its value in the one not visible in the first thirteen cards.Its suit is determined by inspecting whatever suit is not visible from the appropriate clump of four that it's in, in this case the ninth to twelth.

The Gilbreath Shuffle Set-Up

The classic Gilbreath shuffle principle applies to a prearranged packet of cards, cycling $a_0, a_1, \ldots, a_s$, over and over, let's say m times.

E.g., Black and Red cards alternating, say 10 times over.

E.g., Clubs, Hearts, Spades, Diamonds (CHaSeD), cycling 6 times over.

E.g., Ace–King, cycling 3 times over.

More commonly, an entire deck us used, so that we'd have 26, 13 or 4 or the cycles just mentioned.

The Gilbreath Shuffle Principle

Start with a prearranged packet of cards, cycling $a_0, a_1, \ldots, a_s$, over and over, let's say m times.

If any number of cards—traditionally about half of the packet—are dealt to a pile on the table, thus reversing their order, and the resulting two piles are are riffle shuffled together, it turns out that each set of s cards from the top down must consist of one each of $a_0, a_1, \ldots, a_s$, in some order.

The $m = 2$ version was published in *The Linking Ring* in 1958, and the any m version followed there in 1966. Popularised by Martin Gardner.

E.g., applied to the 20 cards cycling Black/Red, we get a packet so that the cards in positions 1–2, 3–4, 5–6, etc, all contain one card of each colour, in some order.

(There *can* be consecutive Red or Black cards, but only in positions 2–3, 4–5, etc, and certainly no more than two in a row of the same colour.)

E.g., applied to the 24 cards cycling CHaSeD, we get a packet so that the cards in positions 1–4, 5–8, etc, all contain one of each suit, in some order.

(Likewise, at most two cards of the same suit can occur together, but only in positions such as 2–5 or 3–6 or 4–7, etc.)

Norman Gilbreath–no mere magician

"While at the RAND Corporation he worked on optimization techniques (linear programming, dynamic programming, network routing), neural network simulation, modifiable compilers, simulating a psychiatric interview, symbiotic research."

The Gilbreath Conjecture about primes (1958)

"Made contributions (conjectures, theorems, principles, articles, books, courses) in mathematics, computer science, magic, organizational development, and cognitive development."

"The Gilbreath Conjecture is easy to state but even though the great number theorist Erdős believed it was true, he also believed it would take about 200 years to prove."

See: *Processing process: The Gilbreath conjecture* (J. of Number Theory, 131, 2436–2441, Dec 2012)

Gilbreath Shuffling versus Riffle Shuffling

One of the piles used in a Gilbreath shuffle is formed by dealing out cards from a prearranged cyclic packet, thus reversing their order.

This step can be skipped for alternating packets: simply split the packet in two and riffle/rosette:

Case 1: If the split resulted in two piles with top (or bottom) cards of different types, then all is well.

Case 2: Otherwise, simply move one card from top to bottom (or vise versa) post shuffle and all is well.

You don't need to know which case applies. . .

The significance of riffle shuffling two piles

The cards within each original "half" maintain the same order relative to each other, after the shuffle.

Think of riffling a Red Ace–King into a Black Ace–King. If the Red cards were then extracted, they would still be in order, as would the Black cards left behind.

There are only $\frac{26!}{13!13!}$, or about 10 million, possible results for such a riffle shuffled half deck of cards, starting with two quarter decks.

That's a very small number compared to 26!

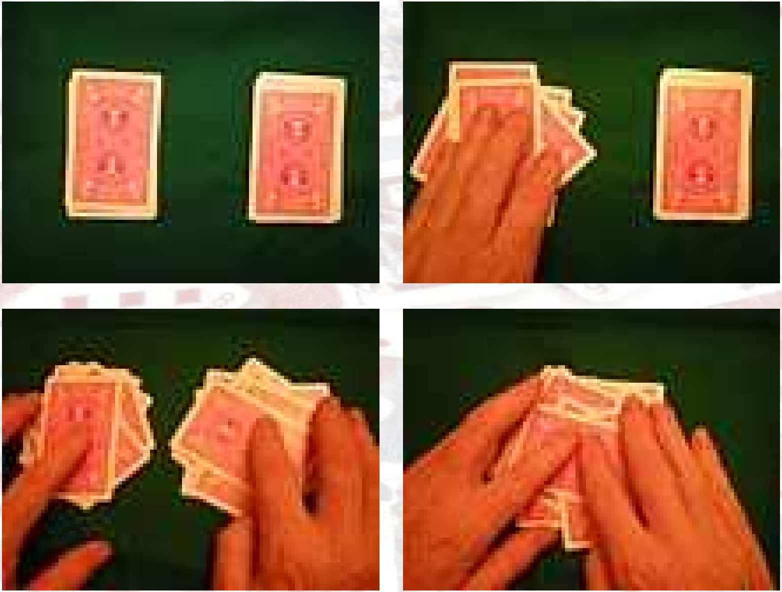
A rosette by any other name

In many ways, riffle shuffling is equivalent to the signature rosette shuffling of Swedish magic maestro Lennart Green:

Take a packet of cards and cut or split it into two piles on a table. (Earlier, the second pile was got by dealing out about half of the given cards.)

Use the fingers to “twirl” each pile into a rosette.

Finally, push these two together, and square up the resulting packet.



Why rosette shuffle?

The packet has effectively been riffle shuffled.

This works well with small packets, where riffle shuffles sometimes present physical challenges.

And it works well for those with smaller hands, including children.

It works for larger packets, provided that a rough surface is used. Then a lot of twirling should be done, to ensure that this action reaches the cards at the bottom of each pile.

References

Mathematical Card Tricks, AMS Online *(October 2000)*

Card Colm, MAA Online *(bimonthly 2004-2014)*

Several MAA **Math Horizons** articles

Chapters in MAA books:

- **Expeditions in Mathematics (The Second Book of BAMA Talks)** *(2011)*
- **The Edge of the Universe—Celebrating Ten Years of Math Horizons** *(2006)*

Chapters in A.K. Peters books:

- **Mathematical Wizardry for a Gardner** *(2009)*
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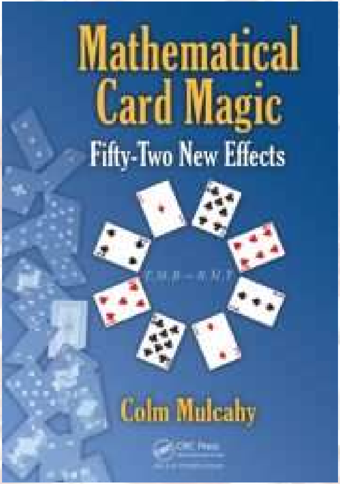
- **A Lifetime of Puzzles** *(2008)*
- **Puzzlers' Tribute: A Feast for the Mind** *(2002)*.

Cool, Colm & Collected



Mathematical Card Magic:
Fifty-Two New Effects

AK Peters/CRC Press, 2013.

Hardback, full colour, 380 pages.

13 main chapters, each with 4 effects.

Largely original material.

Endorsed by Max Maven, Ron Graham, Art Benjamin & Lennart Green.