The Best Friend Mathematics Ever Had

A tip of the hat to the popular mathematics writer Martin Gardner.

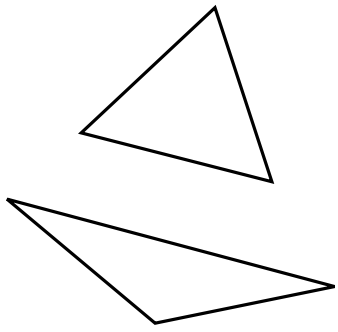
Colm Mulcahy and Albert Goetz

"Warning: Martin Gardner has turned dozens of innocent youngsters into math professors and thousands of math professors into innocent youngsters."
—Perci Diaconis, professor of statistics and mathematics, Stanford University (Gardner 2001)

"Gardner understood instinctively that mathematics was bigger, richer and more fun than what was allowed in class."
—Jordan Ellenberg, professor of mathematics, University of Wisconsin–Madison (Ellenberg 2013)

Can you solve these three brainteasers? The answers may well surprise you; figuring out why they are correct leads to delightful aha! moments.

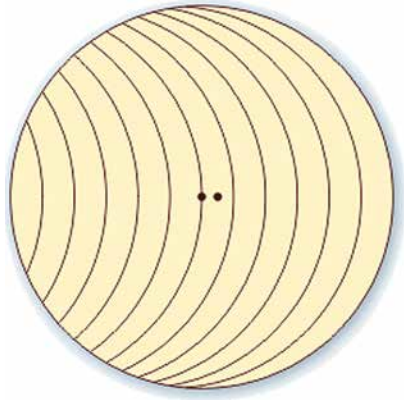
(Answers to these three brainteasers can be found at the end of the article.)



Is the area of the triangle on the top larger or smaller than the area of the triangle on the bottom? The top triangle has sides 5, 5, 6, and the triangle on the bottom has sides 8, 5, 5.

CREDIT TK

How much does a fish weigh if it weighs 1 pound plus half its own weight?



Can you tell which one of the two dots in the illustration is the true center of the circle?

MARVIN GARDNER PHOTOGRAPH BY ELLIOTT ERWITT / MAGNUM PHOTOS
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The kinds of aha! moments that result from solving these challenges for ourselves were held in high estimation by the person who arguably made the greatest twentieth-century contribution to getting people of all ages excited about mathematics. Amazingly, he never took a formal class in calculus or, indeed, anything more advanced than what your students are studying at the moment. That man was the legendary writer Martin Gardner (1914–2010), the best friend mathematics ever had.

A YEAR-LONG CELEBRATION

October 21, 2014, marks the centennial of Gardner's birth and is the centerpiece of celebrations of his life and legacy. Gardner-themed "Celebration of Mind" (celebrationofmind.org) events are held every year in and around his birthday at hundreds of locations all around the planet. They are free and open to all, and everyone is encouraged to attend or host one. They range from small and informal to large—perhaps involving hundreds of participants—and structured. Teachers in particular will find that such events can be tailored to provide a great opportunity to motivate students. The generous resources online are being expanded for this special year.

Many additional activities for stimulating students' interest were made available earlier this year when Mathematics Awareness Month (MAM) was rolled out. Its theme—"Mathematics, Magic, and Mystery"—echoes the title of one of the more than 100 books that Gardner left us. The MAM 2014 materials (mathaware.org/mam/2014/), which will continue to be available for teachers and students indefinitely, relate to thirty fun topics inspired by Gardner's writings, especially his long-running Mathematical Games columns for *Scientific American*.

Teachers are encouraged to investigate the MAM 2014 site for many more games and puzzles that will captivate their students and teach them some mathematics at the same time. Each activity page is introduced by an enticing short video, and further material is provided for those curious to learn more. Topics range from magic squares, Fibonacci numbers, and Möbius bands to juggling, card shuffling, and mathematical magic. Not to mention the ever-popular hexaflexagons (puzzles.com/hexaflexagon), in which strips of paper are folded into hexagonal shapes that can be flexed (turned inside out) to reveal many different faces. "As a kid, I was a hexaflexagon freak all because of that fortuitous first [*Scientific American*] column," recalls Susan Schwartz Wildstrom, of Walt Whitman High School in Bethesda, Maryland. "As a teacher, whenever I teach kids to make them now, they have another burst of popularity. Gardner was definitely a father of 'math for the fun of it all'!" (martin-gardner.org/Testimonials.html#Testi06).

Two "best of" compilation books of those *Scientific American* columns have been issued in recent years: *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems* (Gardner 2001) and *The Colossal Book of Short Puzzles and Problems* (Gardner 2006a); in addition, the Mathematical Association of America put out *Martin Gardner's Mathematical Games: The Entire Collection of His Scientific American Columns*, a single, comprehensive, searchable CD-Rom (Gardner 2006b). Also, *Scientific American* "Mathematical Games" flashcards are available in two separate sets of size 48 (Gardner 2004; Gardner 2006c), each card having a puzzle or brainteaser on one side and its solution on the back. Every mathematics teacher should have these items.

SERIOUSNESS AND FRIVOLITY

Nothing that we know of will excite a student about mathematics more than Gardner's puzzles and mathematical writings, which can also be explored (along with Gardner's many other interests) at the new official Martin Gardner website (martin-gardner.org). About forty of the books Gardner wrote focused on mathematics and puzzles, and more than sixty focused on short fun puzzles (such as the second and third brainteasers given earlier) linked from the puzzles.com website (puzzles.com/puzzleplayground/Authors/MartinGardner.htm). Both his substance and his style—because Gardner had an inviting, easy-to-understand way of presenting a problem and its solution—make his puzzles a catalyst for learning mathematics.

All teachers feel the pressure of sticking to the curriculum and preparing students for inevitable tests, but perhaps there is measurable value to carving out a little time to have a little fun too. Some teachers have experimented with setting aside five minutes in the middle of class to go off-topic and get students' attention with something unexpected and refreshing like a Gardner puzzle. Students look forward to it, seem invigorated by it, and are more alert throughout the period as a result.

The well-known calculus textbook author Roland Minton, professor of mathematics at Roanoke College in Virginia, comments, "Over the last few years I have been inspired and invigorated by Martin's books and columns, which have been a wonderful gift for a 30+ year professor. Taking time, both in and out of class, to share my latest favorite Gardner fact is the best way I know to get students interested in mathematics" (martin-gardner.org/Testimonials.html#Testi52).

As Gardner acknowledged at the start of the

introduction to his book *Mathematical Carnival*, "A teacher of mathematics, no matter how much he loves his subject and how strong his desire to communicate, is perpetually faced with one overwhelming difficulty: How can he keep his students awake?" (1989, p. 1). After delivering a failing grade to the "new math," which had by that time run its course, Gardner wrote, "The best way, it has always seemed to me, to make mathematics interesting to students and laymen is to approach it in a spirit of play" (1989, p. 1).

In that spirit, consider some of Gardner's favorite questions: Why does a mirror reverse left and right but not up and down? Why do the sun and the moon appear to be the same size from our vantage point in the solar system? One of his most famous brainteasers, from an era before calculators, is this:

> Two missiles speed directly toward each other, one at 9,000 miles per hour and the other at 21,000 miles per hour. They start 1,317 miles apart. Without using pencil and paper, calculate how far apart they are one minute before they collide. (Colliding Missiles, problem 4.11, Gardner 1994, p. 87)

Try to solve this before we give the solution.

Any luck solving this? Well, a little physics instinct suggests combining the speeds, so that it's as if one missile is stationary and the other is barreling toward it at 30,000 mph. However, that 1,317 figure is worrisome. Surely working with that, under the given conditions, is going to involve some tricky mental gymnastics? Then comes the aha! moment: Simply note that a minute is 1/60 of an hour, and here distance = speed • time. Hence, in any given minute, the distance (in miles) between the missiles decreases by 30,000 • (1/60) = 500. So they must be 500 miles apart a minute before they collide. We never used the fact that they start 1,317 miles apart, because, as it turns out, it had no bearing on the question posed.

At this point, some readers may be feeling let down or even cheated. Our students might well feel the same way if they were asked to tackle this puzzler. Is it a trick question? Yes, if we consider it wise always to provide no more and no less information than is required to solve every problem (although arguably such spoon-feeding ultimately does a disservice to the learner). No, if we wish to prepare students for real life, with its information overload, where being able to distinguish the essential from the inessential is a key skill to be acquired. Perhaps instead of shunning problems posed in this way, we would be wiser to include more of them in our teaching. What kind of problem solvers are we

He brought more mathematics to more millions than anyone else.

—Richard Guy, emeritus professor of mathematics, University of Calgary; quoted in the foreword to *The Mathemagician and Pied Puzzler: A Collection in Tribute to Martin Gardner*, edited by Elwyn Berlekamp and Tom Rodgers (AK Peters, 1999)

Martin Gardner took no mathematics courses after high school. He attempted to learn calculus in college but failed.

—Steven Krantz, professor of mathematics, Washington University in St. Louis; quoted in *Notices of the American Mathematical Society*, vol. 58, no. 3, March 2011, ams.org/notices/201103/rtx110300418p.pdf

Martin offers everybody (not just mathematicians) creative refuge for the imagination.

—Isaac Asimov, popular science and science fiction author

For those of us who have tried to make mathematics accessible to a wider audience, there is one giant who towers above everybody else: Martin Gardner.

—Keith Devlin, executive director, H-STAR Institute, Stanford University, and NPR's "The Math Guy"

What Martin Gardner has done is of far greater originality than work that has won many people Nobel Prizes.

—Douglas R. Hofstadter, author, *Godel, Escher, Bach: An Eternal Braid* (Basic Books, 1979)

developing if we never allow students practice in determining what information they need to use?

Ironically, despite Gardner's considerable reputation for leading so many people to mathematics, he had minimal training in the subject—just the algebra, trigonometry, and geometry of an Oklahoma high school graduate of 1932. Although he originally planned to study physics at California Institute of Technology, he graduated from the University of Chicago in 1936, having majored in philosophy.

By the late 1950s, Gardner was the author of a highly successful *Scientific American* column on recreational mathematics. Many people cite the column (which ran for twenty-five years) or the resulting books of compilations as a primary influence on their decision to enter the field of mathematics. Gardner introduced generations of readers to the Soma cube; origami; rep-tiles; tangrams; polyominoes; the art of M. C. Escher; the $3n + 1$ problem; Conway's game of Life; Fermat's last theorem; the four-color map problem; RSA cryptography; fractals; and a host of intriguing paradoxes.

As a member of that rare breed of self-taught mathematicians, Gardner welcomed interactions with and contributions from amateurs as well as professionals. One spectacular result was the case of a middle-aged housewife named Marjorie Rice, who had no more mathematics background than Gardner. On reading one of his Mathematical Games columns, she was inspired to discover original geometric patterns concerning tiling the plane with pentagons. She communicated her findings to Gardner; as a result, her work was published in 1977 (mathmunch.org/2013/02/25/marjorie-rice-inspired-by-math-and-subways).

In his introduction to *Mathematical Carnival*, Gardner concluded "Surely the best way to wake up a student is to present him with an intriguing mathematical game, puzzle, magic trick, joke, paradox, model, limerick, or any of a score of other things that dull teachers tend to avoid because they seem frivolous. No one is suggesting that a teacher should do nothing but throw entertainments at students. Obviously there must be an interplay of seriousness and frivolity. The frivolity keeps the reader alert. The seriousness makes the play worthwhile" (Gardner 1989, p. 2).

In 1998, in his final, retrospective column for *Scientific American*, "A Quarter Century of Recreational Mathematics," Gardner wrote, "The line between entertaining math and serious math is a blurry one." He continued, "The monthly magazine published by the National Council of Teachers of Mathematics, *Mathematics Teacher*, often carries articles on recreational topics. Most teachers, however, continue to ignore such material. For 40 years I have done my best to convince educators that recreational math should be incorporated into the standard curriculum. It should be regularly introduced as a way to interest young students in the wonders of mathematics. So far, though, movement in this direction has been glacial" (Gardner 1998, p. 68).

How's that for a challenge?

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ANSWERS TO BRAINTEASERS

The answer to brainteaser 1 is short and sweet: no. (Or, perhaps more accurately, neither.) Go back, read the question very carefully, and do the mathematics. Hint: The answer hinges on a particular Pythagorean observation. This problem is by Dick Hess (Hess 2009). (This website reveals all: futilitycloset.com/2012/07/21/tall-and-wide/.)

For brainteaser 2, the fish weighs 2 pounds. (Two halves make a whole, so if the fish's total weight is the sum of 1 pound and half the total weight, the other half must also be 1.) This is "Fish's Weight" (puzzles.com/puzzleplayground/FishsWeight/FishsWeight.htm).

Brainteaser 3, about the true circle center, is "Find the Center" (puzzles.com/puzzleplayground/FindTheCenter/FindTheCenterExp.htm), which we leave to readers to explore further (note that the linked image rotates).

Did you have any aha! moments yet? We certainly hope so!


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